RATE OF HEAT PROPAGATION BY CONDUCTION IN GASES

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A formula giving the rate of heat propagation by thermal conduction in gases as a function of absolute temperature is derived.

It is usually assumed in the theory of heat conduction that thermal energy is propagated instantaneously by the conduction mechanism. This assumption satisfies a large number of technical problems for low velocities of the medium and a low intensity of the heat-transfer processes. With the development of techniques using high-intensity, transient processes, involving high velocities of the heat carriers (coolants), it is usual to assume that the heat is propagated, not infinitely rapidly, but with a certain high, finite velocity. A hypothesis as to the finite rates of heat propagation W_q was proposed in [1], and a formula was given for its calculation

$$W_q = \sqrt{\lambda / c\rho\tau}. \tag{1}$$

It was noted in [2], however, that the experimental measurement of the relaxation time τ was quite impossible with contemporary measuring methods. For practical calculations Eq. (1) is therefore inapplicable. For practical purposes we require a formula expressing the rate of heat flow in terms of known or easily determinable quantities. We set ourselved the task of converting the Lykov formula (1) to a form better suited to technical calculations.

For a steady-state, one-dimensional temperature field, the scalar magnitude of the thermal flux density is

$$q = -\lambda \frac{dT}{dx}. \tag{2}$$

On considering the flow of heat through a layer of stationary gas (as a condition for the contact relationship between the gas and the surrounding medium we must take p = const), the Fourier law of heat conduction may be written in the form [2]

$$q = -\frac{\lambda}{\rho c_p} \cdot \frac{dH}{dx}. \tag{3}$$

In determining the instantaneous velocity of the heat flow, i.e., the velocity at a specified point in space, we must take a layer of stationary gas with as small a width as possible. The smaller we take the width of the gas layer, the more fairly may we consider λ = const, ρ = const, and c_p = const. Then in the steady heat flow (q = const) the enthalpy gradient will be constant, i.e., the enthalpy will vary linearly with x, and we may write

$$\frac{dH}{dx} = \frac{H_2 - H_1}{x_2 - x_1} = -\frac{H_1 - H_2}{x_2 - x_1} = \text{const}. \tag{4}$$

We see from the manner in which the relationship for the thermal conductivity was derived in [3] that the smallest linear dimension may only be twice the mean free path of the molecules, since $x_2 - x_1 = (z + l) - (z - l) = 2l$. For a layer of thickness $2l$, allowing for the foregoing considerations, Eq. (3) may be written in the form

$$q = \frac{\lambda}{2\rho c_p l} (H_{z-l} - H_{z+l}), \tag{5}$$

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from which it is easy to see that the instantaneous (local) rate of heat flow is

$$W_q = \frac{\lambda}{2\rho c_p l}, \tag{6}$$

since the flux density of any quantity equals the product of its volume concentration and velocity.

According to the molecular kinetic theory of gases,

$$\lambda = 1/3\rho l\bar{v}c_v = \eta c_v, \tag{7}$$

where the mean velocity of the chaotic (random) motion of the molecules

$$\bar{v} = \sqrt{\frac{8}{\pi} RT}\,. \tag{8}$$

Equation (8) was obtained for a monatomic gas. In order to allow for the polyatomic state of the molecules, certain authors, for example, Mott-Smith [4], use Eq. (8) in the form

$$\bar{v} = \sqrt{\frac{\gamma}{\gamma - 1} RT}\,. \tag{9}$$

From Eq. (7) we derive the simple theoretical relationship $\lambda/\eta c_V = 1$, which is not confirmed by experiment. This necessitates the introduction of a correcting factor into Eq. (7)

$$S = \lambda/\eta c_v, \tag{10}$$

which may be expressed in terms of the Prandtl number

$$\mathrm{Pr} = \eta c_p/\lambda, \tag{11}$$

namely

$$S_D = \gamma/\mathrm{Pr}. \tag{12}$$

Using Eqs. (7), (9), and (12), Eq. (6) may be reduced to the form

$$W_q = \frac{1}{6\mathrm{Pr}} \sqrt{\frac{\gamma}{\gamma - 1} RT}\,. \tag{13}$$

Equation (13) may easily be used in practical calculations, since it is mathematically fairly simple, while the values of Pr and γ are tabulated for many gases on the basis of experimental data covering a wide temperature range.

A number of research workers, using the molecular kinetic theory of gases, allowing for various components of the specific heat (translational, rotational, and vibrational degrees of freedom) and other physical characteristics of the gas, have derived a variety of relations for the correction factor S. In cases in which they prove accurate enough, these correction factors are mathematically complicated (Mason, Monchik, Saxen, Gambir, and others) and cannot be used in technical calculations. The simplest is that of Aiken

$$S_E = \frac{9\gamma - 5}{4}, \tag{14}$$

and on using this Eq. (6) may be reduced to the form

$$W_q = \frac{9\gamma - 5}{24\gamma} \sqrt{\frac{\gamma}{\gamma - 1} RT}\,. \tag{15}$$

The Aiken correction factor, which has been regarded as satisfactory for some 15 years, gives large errors for certain gases. Table 1 compares the values of W_q calculated by Eqs. (13) and (15).

Equation (13) indicates that W_q depends solely on the nature and temperature of the gas. The thermal flux exists in the presence of a temperature gradient, and of course the rate of heat propagation will diminish along the direction of heat flow. Allowing for the foregoing, the velocity calculated from Eq. (13) should be called the local heat velocity.

Remembering that the velocity of sound $a = \sqrt{\gamma RT}$, Eq. (13) may be modified:

$$W_q = \frac{1}{6\mathrm{Pr}\sqrt{\gamma - 1}} \sqrt{\gamma RT} = \frac{1}{6\mathrm{Pr}\sqrt{\gamma - 1}}\, a \tag{16}$$

TABLE 1. Rate of Heat Propagation by Conduction for Several Gases at Atmospheric Pressure

Gas	Pr	W_g from (13)	W_g from (15)	Deviation, %	B
Argon	0,663	94,2	93,9	—0 ,319	0,307
Helium	0,684	290	297	+2 ,41	0,298
Hydrogen	0,688	476	447	—6 ,09	0,378
Nitrogen	0,705	126	120	—4 ,76	0,372
Oxygen	0,720	116	113	—2 ,59	0,367
Air	0,707	124	119	—4 ,03	0,373
Carbon disulfide	0,778	85,4	82,2	—3 ,75	0,447
Carbon dioxide	0,780	101	102	+0 ,99	0,390
Water vapor	1,06	119	161	+35,3	0,297
Amonia	0,908	138	162	+17,4	0,330
Methane	0,734	175	166	— 5,14	0,406
Sulfur dioxide	0,874	71,9	79,6	+10,7	0,367

Note. 1) All gases taken at 0°C, water vapor at 100°C. 2) Values of γ and Pr taken from [5].

and denoting

$$B = \frac{1}{6\mathrm{Pr}\sqrt{\gamma - 1}}, \tag{17}$$

we obtain

$$W_q = Ba. \tag{18}$$

We see from Table 1 that $B < 1$, and hence the velocity of heat flow is always lower than the velocity of sound.

We see that the relation for the heat velocity in a stationary gas will apply to a gas flow also if we use a system of coordinates connected to the center of mass of the element of gas. In practice many problems are encountered in which it is required to determine the velocity of heat propagation relative to the stationary walls of a channel through which the gas flow is passing. Transfer of the coordinate system to the stationary wall gives

$$W_{q_W} = W \pm W_q. \tag{19}$$

In an accelerating gas flow, the temperature diminishes on passing along it ($dW > 0$ but $dT < 0$), the gas and heat flows coincide in direction, and in Eq. (19) the plus sign has to be taken. In a decelerating gas flow, the temperature rises along the flow ($dW < 0$ but $dT > 0$), the thermal flux opposes the gas flow, and in Eq. (19) the minus sign has to be taken. When the velocity of the gas flow reaches that of the heat flow, the velocity of the heat flow relative to the wall vanishes, since in this case $W = W_q = Ba$ and $W_{qw} = 0$. It also follows that in a decelerating gas flow with $M \geq B$ it is impossible for heat to propagate by conduction against the gas flow.

The chief problem in the analytical theory of heat conduction is that of finding the temperature field. Knowing the temperature field, it is easy to use (13) in order to determine the field of conductive heat flow velocities.

NOTATION

W, W_q, W_{qw}	are the velocities of the gas flow, the heat flow relative to the center of mass of the gas element, and the heat flow relative to the stationary walls of the channel, m/sec;
τ	is the relaxation time, sec;
ρ	is the mass density, kg/m^3;
c, c_v, c_p	are the gravimetric specific heat and specific heat at constant volume and pressure, J/kg • deg;
λ	is the thermal conductivity, W/(m • deg);
q	is the heat flux density, W/m^2;
T	is the absolute temperature, °K;
H	is the volumetric enthalpy concentration, J/m^3;
x	is a coordinate, a linear dimension, m;

l	is the mean free path of the molecules, m;
$\bar{v}$	is the mean velocity of random motion of the molecules, m/sec;
R	is the gas constant, J/(kg · deg);
S	is the correction factor;
a	is the velocity of sound, m/sec;
$M = W/a$	is the Mach number;
γ	is the ratio of the specific heats;
η	is the viscosity, N · sec/m^2.

LITERATURE CITED

1. A. V. Lykov, Heat Conduction and Diffusion [in Russian], Gizlegprom (1941).
2. A. V. Lykov, Theory of Heat Conduction [in Russian], Vysshaya Shkola (1967).
3. A. G. Shashkov and T. N. Abramenko, Heat Conduction of Gas Mixtures [in Russian], Énergiya, Moscow (1970).
4. Mott-Smith, "Solution of the Boltzmann equation for a shock wave," in: Mechanics [Russian translation], No. 1 (1953).
5. S. S. Kutateladze and V. M. Borishanskii, Handbook on Heat Transfer [in Russian], Gosenergoizdat (1959).